Basic Principles of Magnetic Resonance

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Contents

1 Introduction

The primary purpose of this lecture is to provide an overview of the basic principles involved in the process of obtaining a nuclear magnetic resonance signal. The following lectures will discuss how this signal can be used to generate images that carry information about brain function.

The lecture begins with a short and incomplete history of the developments that led to the discovery of key imaging concepts. The third and largest section covers the basic physics that explain the origin and some of the properties of the nuclear magnetic resonance signal. Finally, some relevant reference texts and articles are listed.

These lecture notes are not complete and should be taken as a guide. It is expected that students will complete the suggested reading for a a more complex coverage of the topics discussed.

2 MRI: a brief historical background

Magnetic Resonance Imaging, or MRI, stems from the application of nuclear magnetic resonance (NMR) to radiological imaging. The adjective 'magnetic' refers to the use of an assortment of magnetic fields and 'resonance' refers to the need to match the (radio)frequency of an oscillating magnetic field to the 'precessional' frequency of the spins of some atomic nucleus (hence the 'nuclear') in a tissue molecule.

The concept of nuclear magnetic resonance started with the discovery of the spin nature of the proton followed by the study of the interaction of this spin with a magetic field. The phenomenon of magetic resonance was first applied for studying the chemistry and structure of solids and liquids. The possibility of using MR for the study of living tissue sparked interest in the development of bio-medical applications, particularly when it was proved that abnormal and normal tissues could be distinguished using MR. Nearly four decades passed before MR was successfully employed in medical imaging.

Below is a brief and incomplete list of some of the milestones in the development of MRI as we use it today.

- **1922, Otto Stern & Walter Gerlach**
  Experimental observation of spin quantization in electrons: Stern and Gerlach passed a beam of silver atoms through an inhomogeneous magnetic field to study the magnetic properties of the electron. The silver atoms were in their ground or equilibrium state, which means that the net electric charge was zero and that they had a single unpaired electron in the outer electron orbit. At that time the expected result was that the beam of silver atoms should have a smooth distribution around the center because the magnetic moment of the atom (due only to the unpaired electron) should feel a net force in the inhomogeneous magnetic field and because all possible orientations of the magnetic moment should be in principle possible. However, the result was that the beam was split into two clearly separate components of equal intensity. This result was later explained by Uhlenbeck and Goudsmit (1925, 1926), who proposed that the electron had an intrinsic magnetic moment, or spin, with only two possible orientations, thereby introducing the concept of spin quantization.

- **1937, Isidor I. Rabi** (Nobel Prize in Physics, 1944)
  NMR phenomenon in molecular beams: Radio-Frequency (RF) energy is absorbed by atomic nuclei within samples placed in a strong magnetic field. For the absorption to be efficient the RF must have a special frequency called resonance frequency or Larmor frequency. The Larmor frequency is defined by the magnetic field strength and the atomic nuclei.

- **1945, Felix Bloch & Edward M. Purcell** (Nobel Prize in Physics, 1952)
  NMR phenomenon in solids: Bloch & Purcell, independently, where the first to demonstrate the NMR phenomenon in bulk materials.

- **1949, Norman F. Ramsey** (Nobel Prize in chemistry, 1989)
  Chemical shift theory: atomic nuclei in different chemical environments can be identified as a result of a small change in resonance frequency caused by the electron cloud of the molecule. A molecular system can be thus described by its absorption spectrum: continuous wave magnetic resonance spectroscopy

was born. The sensitivity of the experiment was low: each resonance frequency (i.e., each nuclei species) was separately excited. To achieve enough signal-to-noise ratio many excitations were necessary for averaging, making the experiments extremely slow.

- **1971, Raymond V. Damadian**
  Tumor detection is possible using NMR: Cancerous tissue in rats exhibited dramatically prolonged NMR relaxation times. Relaxation times of normal tissues also vary significantly, though less dramatically than cancer tissue. With these findings Damadian conceived and originated the application of magnetic resonance technology to medical uses, including whole-body scanning and diagnostic imaging.

- **1972, Paul C. Lauterbur**
  MR image principle: the shift in resonance frequency resulting from the imposition of a magnetic field gradient can be used to generate a two-dimensional spatial distribution of protons in a water sample.

- **1976, Peter Mansfield**
  Echo Planar Imaging (EPI): this is the most common fast MRI technique. It was not until the mid 80s, however, that hardware improvements made possible the acquisition of EPI images of clinical use in the order of 100ms. There are now faster imaging methods, but these methods have limited applications due to their inherent low signal to noise ratio.

- **1980s - present**
  During this period both MRI methods and hardware were further developed to produce a tool that is nowadays used for a wide variety of human and animal studies. MRI is used not only to produce anatomical images with below mm spatial resolution, but also to study blood flow, perfusion, diffusion, as well as the function of the brain and of other organs. The development of faster imaging techniques with improved properties regarding contrast, resolution and signal to noise ratio continues.

3 A MR Experiment

3.1 Overview

Several steps are involved in the production of a MR signal. These may be summarized as follows:

i) In dealing with human subjects, the subject's safety has the highest priority. Therefore, prior to the examination the subject to be imaged must be screened to make sure that it is safe for him/her to be exposed to the electromagnetic fields used for the MR procedure.

ii) Once in the magnet, randomly oriented tissue atomic nuclei are aligned by a powerful, uniform magnetic field giving rise to a macroscopic magnetization in an equilibrium state. Several atomic nuclei (13O, 19F, 23Na, 39K, etc.) could in theory be imaged with MR. However, MR is primarily applied to imaging hydrogen protons in water, for two reasons: high sensitivity for its MR signal and high natural abundance in biological tissue.

iii) The equilibrium magnetization can be disrupted (excited) by absorbing energy from properly tuned radio-frequency pulses. After the excitation the magnetization recovers to its original equilibrium state through relaxation processes. In this process radio signals are emited that can be detected: the MR signal. Tissue contrast (i.e., differences in signal intensity) develops as a result of the different rates at which nuclei recover to the equilibrium state.

3.2 Can we scan the subject?

The purpose of this section is to briefly discuss some MRI safety issues and to emphasize that in principle not every person can undergo an MRI experiment. Human safety issues during MRI examinations will be covered in more detail on a later lecture by Randy Gollub.

All individuals, including patients, volunteer subjects, visitors, MR health care providers, and custodial workers, must be thoroughly screened by qualified personnel before being exposed to the static, gradient, or radio frequency electromagnetic fields of the MR system. The initial screening process should involve

completion of a questionnaire that is specifically designed to determine if there is any reason that the individual would have an adverse reaction to the electromagnetic fields used for the MR procedure.

The following is a list that includes some of the important aspects to be considered in the screening process:

- *Ferromagnetic objects* will be subject to attraction forces in presence of an external magnetic field. Therefore, extreme care has to be taken to avoid that any ferromagnetic objects enter the magnet room. Subjects having ferromagnetic (e.g., cast iron, steel, etc.) devices, biomedical implants or objects should not undertake an MR examination because of the risks associated with the movement or dislodgement of these objects. In addition, even if the forces are small, active biomedical implants may not function properly in the MRI field due to interference between the field and the electronic components of the implants.
- *Non ferromagnetic objects* (e.g., aluminum, beryllium, copper, lead, magnesium, nickel, gold, silver, titanium) will not be subject to forces in the MR environment (in this sense they are safe) but may induce distortions in the MR images. Artifacts are seen typically as local regional distortions of the image and /or as signal void. These artifacts result from eddy currents (Faraday induction's law) that can be generated in the objects by the gradient magnetic fields used for MRI. The induced eddy currents generate a small but non-neglible magnetic field that distorts the local magnetic field. Such distortions disrupt the relationship between position and magnetic field strength that is crucial for accurate image reconstruction. The eddy currents may also induce minor temperature elevations during the MR procedure.
- *Claustrophobia/anxiety:* as many as 20% of individuals attempting to undergo an MR procedure cannot complete it due to claustrophobia or other similar sensations. These sensations originate from one or more factors involved in the MR procedure, including the confining dimensions of the interior of the MR system, the distress related to the restriction of movement, the prolonged duration of the examination, the acoustic noise generated by alternating currents in the magnetic field gradient coils (see below).
- *Movement:* MR procedures with subjects who cannot remain still during the study may result in distorted images and/or corrupted dynamic studies in which various images are acquired sequentially, such as in functional MRI. For this reason, if possible, it is adviced not to scan subjects who have an illness condition which will prevent them from remaining still during the MR examination (e.g., persistant cough, sneezing, etc.).
- *Acoustic protection:* subjects must use earplugs as means of preventing problems associated with the various types of acoustic noises produced during the operation of the MR system. The primary source of acoustic noise associated with MR procedures is the switching magnetic field gradients. As it will be explained in the following lecture, these gradients are used to encode spatial information in the MR signal. The noise occurs during the rapid alternations of currents within the gradient coils. These currents, in the presence of a strong static magnetic field of the MR system, produce significant forces that act on the gradient coils. Acoustic noise, manifest as a loud tapping, is produced when the forces cause motion or vibration of the gradient coils as they impact against their mountings. For this reason, if possible, subjects who have an illness (e.g., ear infection) that prevents them from being comfortable with earplugs should not be scanned.

For information of scenarios you definetely want to avoid take a look at:
http://www.simplyphysics.com/flying_objects.html
http://www.nytimes.com/2001/07/31/nyregion/31MRI.html

3.3 The subject goes into the magnet

In this section we will see that the purpose of placing the subject into the magnet is to induce a macroscopic magnetization that we can later measure. The origin of this magnetization will be described as well as its dynamics in a magnetic field. This will set the basis for the next section, which will address how we can interact with and thereby measure the magnetization by taking it out from its equilibrium state.

3.3.1 Equilibrium magnetization $\mathbf{M_0}$

For the purpose of generating a MRI it is necessary to place the the subjet in a magnetic field that has particular properties: it is powerful (typically ∼30,000 times stronger than the earth's magnetic field), it is static (it does not change with time, at least ideally) and it is spatially uniform (ideally). This field induces within the subject's tissues a net macroscopic nuclear magnetization. Why does this happen? To understand this let us first consider a single hydrogen nucleus and then a group of nuclei.

A hydrogen nucleus (i.e., a single proton) has two important properties: an electric charge and a spin. A fundamental physical law tells that a moving electrical charge possesses a magnetic field. Thus, the hydrogen nucleus has a magnetic field which can interact with an external magnetic field. The distribution of the proton's magnetic field is that of a point dipole and is described by a (dipolar) magnetic moment vector (i.e., it has a magnitude and a direction).

The details of the interaction between the nuclear magnetic moment and an external field must be explained using quantum mechanics. It can be shown that when a proton is placed in an external field $\mathbf{B_0}$ there are only two possible energy states available for the system: a low energy state (magnetic moment aligned parallel with $\mathbf{B_0}$), and a slightly higher energy state (magnetic moment aligned anti parallel with $\mathbf{B_0}$).

The energy difference ΔE between these two states is given by the following equation:

$$\Delta E = \hbar\omega_0 = \gamma\hbar\mathbf{B_0} \tag{1}$$

where γ is known as the gyromagnetic ratio of the nucleus (it depends on the size and shape of the nucleus), $\hbar$ is Planck's constant (divided by 2π) and ω_0 is the frequency of the electromagnetic radiation necessary to induce a transition between the two states. Also, ω_0 is the frequency at which the magnetic moment of the nucleus is precessing about the direction of the main field $\mathbf{B_0}$. In water, the hydrogen proton has a γ value of roughly 2.68×10^8 rad/s/Tesla, so that for a for a 3 T field, for example, the spins precess at a radiofrequency of 127.8 MHz, which is in the FM range for radio broadcasting.

Let us now consider an ensemble of non interacting hydrogen nuclei. Thus, the system of interest is a spin in thermal contact with the rest of a set of N spins and with the background lattice all at temperature T. When these nuclei are in thermal equilibrium and in an external field $\mathbf{B_0}$, some of them (N_+) will be in the low energy state and some of them (N_-) will have enough kinetic energy to be in the high energy state ($N = N_+ + N_-$). It can be shown that in thermal equilibrium the statistical distribution of the spin states will be given by the Boltzmann distribution,

$$\frac{N_-}{N_+} = \exp\left(-\frac{\Delta E}{kT}\right) \tag{2}$$

where k is the Boltzmann constant and T is the absolute temperature of the spin system. This expression implies that, at thermal equilibrium, there will be more spins in the lower energy state than in the higher one. Since the magnetic moment of the spins in the low energy state is equal in magnitude but with the opposite direction to the magnetization of the spins in the high energy state, there will be some cancellation. Thus, it is the difference between the two populations (or spin excess in the lower energy state) that matters. This difference will give rise to a net macroscopic magnetization $\mathbf{M_0}$ aligned in parallel to the external field $\mathbf{B_0}$. In the plane perpendicular to $\mathbf{B_0}$ (also called the transverse plane) there will be no net magnetization because the transverse projections of all the precessing spins are randomly distributed, giving a zero net component.

The extreme smallness of the quantum spin energy compared with the thermal energy means that the fraction $\hbar\omega_0/(kT) << 1$. Therefore, the two energy states are nearly equally populated with only a small majority in the lower energy state. At a typical field strength of $\mathbf{B_0}$ =1.5 T and at room temperature a simple calculation shows that the excess of protons in the energetically favorable state is about 5 out of every million protons. It might be guessed that with this small spin excess no significant signal would be detected at room temperature. However, there are Avogadro numbers of protons ($6\ 10^{23}$)in a few grams of tissue. This leads to measurable NMR effects, to be described next.

3.3.2 Dynamics of the magnetization

In the previous section we introduced the concept of equilibrium magnetization when the sample or subject is in the static main field $\mathbf{B_0}$. In this section the general equation of motion of the magnetization in an arbitrary external magnetic field will be introduced. This simple equation can be later use to describe how external magnetic fields can be used to interact with the magnetization and thus help us to generate a MR signal.

Assuming that the spins do not interact with each other (they are uncoupled, which is a reasonable assumption for fluids, i.e. tissue), we can describe the dynamics of the macroscopic magnetization using a classical mechanical formalism. To obtain the equation of motion, the strategy is to model the behaviour of a single nucleus magnetic moment and then add up all the spins of the sample to form the net magnetization vector.

Equation of motion for a single nucleus
To describe the spin in the magnetic field using classical physics we need 3 equations:

- The following fundamental equation establishes a connection between the mechanical moment and the angular momentum of the rotating atomic nucleus.

$$\mathbf{T} = \frac{d\mathbf{L}}{dt} \tag{3}$$

  which expresses that the rate of change of the spin (angular momentum) $\mathbf{L}$ defines the torque $\mathbf{T}$ on the nucleus.

- Next we need to know the connection between the spin and the magnetic moment of the nucleus.

$$\mu = \gamma \mathbf{L} \tag{4}$$

  where μ is the magnetic moment of the nucleus and γ its gyromagnetic ratio.

- At last the equation which actually formulates the interaction between an external magnetic field and the magnetic moment of a single nucleus

$$\mathbf{T} = \mu \times \mathbf{B}_{ext(t)} \tag{5}$$

  with $\mathbf{B}_{ext(t)}$ an arbitrary magnetic field.

That's it, with these three equations it is possible to eliminate $\mathbf{T}$ and $\mathbf{L}$ to express the equation of motion of the magnetic moment of a single nucleus in the field $\mathbf{B}_{ext(t)}$:

$$\frac{d\mu_{(t)}}{dt} = \mu_{(t)} \times \gamma \mathbf{B}_{ext(t)} \tag{6}$$

Equation of motion for the magnetization vector
To get the magnetization vector we simply have to add up all the tiny atomic moments. Using the assumption that the interaction of the nuclei magnetic moments can be neglected (which is a reasonable assumption at room temperatures), we can write down the magnetization vector as

$$\mathbf{M} = \mu_0 + \mu_1 + \mu_2 + ... = \sum \mu_i \tag{7}$$

The actual total number of magnetic moments depends on the size of the sample, but lies in the magnitude of 10^{23} spins. Differentiating $\mathbf{M}$ after t yields

$$\frac{d\mathbf{M}_{(t)}}{dt} = \mathbf{M}_{(t)} \times \gamma \mathbf{B}_{ext(t)} \tag{8}$$

The reason for considering a general $\mathbf{B}_{ext(t)}$ will become clearer in the next section where we discuss the process of exciting the magnetization using an oscillating rf pulse. The equation of motion for the magnetization states that at any instant of time t the magnetization $\mathbf{M}_{(t)}$ is precessing about the field $\mathbf{B}_{ext(t)}$ at the frequency $\omega_{(t)}$ given by the Larmor equation,

$$\omega = \gamma B_{ext} \tag{9}$$

The geometrical description of the equation of motion for the magnetization is therefore a vector $\mathbf{M}$, with its origin fixed at the origin and having a rotational motion around the axis defined by the vector $\mathbf{B_{ext}}$. The angular frequency of the precession of $\mathbf{M}$ about $\mathbf{B_{ext}}$ is given by $\omega = \gamma B_{ext}$.

At thermal equilibrium the solution to the equation of motion for the magnetization is trivial (and we know it already). The conditions at equilibrium are: $\mathbf{B}_{ext(t)} = \mathbf{B_0}$ and $\mathbf{M}_{(t=0)} = \mathbf{M_0}$ (no initial transverse magnetization). Since $\mathbf{B_0}$ and $\mathbf{M_0}$ are parallel then the magnetization is constant ($d\mathbf{M}/dt$ =0) and aligned with the main field: $\mathbf{M}_{(t)} = \mathbf{M_0}$.

3.3.3 Rotating coordinate system

Remember that the goal of the MR experiment can be summarized in three steps: i) place the sample in a strong and uniform magnet to generate a macroscopic magnetization, ii) excite the magnetization by applying an external rf pulse and iii) measure the signal generated by the rotating magnetization. The process of exciting the magnetization and obtaining an MR signal will be discussed in the following sections. To facilitate that description it will useful to introduce at this point the so-called rotating coordinate system.

In order to describe the dynamics of the magnetization $\mathbf{M}_{(t)}$, it is a useful convention in NMR to define a coordinate system such that its z-axis is parallel to the external static field $\mathbf{B_0}$. In such a system, and at equilibrium, the vector $\mathbf{M_0}$ is aligned along the z-axis. In a general case in which the total magnetic field $\mathbf{B}_{ext(t)}$ is composed of a constant field term $\mathbf{B_0}$ and an applied time varying field $\mathbf{B}_{1(t)}$ such a description would be difficult to visualize since the magnetization will try to precess about the net field $\mathbf{B}_{ext(t)} = \mathbf{B_0} + \mathbf{B_{1(t)}}$.

The description of the dynamics of $\mathbf{M}$ can be significantly simplified by introducing the use of a coordinate system rotating about $\mathbf{B_0}$ at a frequency ω_0 (spins which have this resonance frequency are sometimes referred to as on-resonance spins). It can be proven that in the rotating frame the equation of motion of the magnetization takes a particularly easy form,

$$\frac{d\mathbf{M}_{(t)}}{dt} = \mathbf{M}_{(t)} \times \gamma \mathbf{B'}_{1(t)} \tag{10}$$

where $\mathbf{B'}_{1(t)}$ is the arbitrary time varying field $\mathbf{B}_{1(t)}$ expressed in the rotating frame. This equation says that in the rotating frame the magnetization of the spins on resonance will precess about the field $\mathbf{B'}_{1(t)}$ with a frequency $\omega_1 = \gamma B'_1$.

Thus, in the rotating frame and when only main field is applied ($\mathbf{B'}_1$=0), the magnetization $\mathbf{M}$ will be static for on-resonance whereas it will precess about the main field for off-resonance spins. This concept will be useful to describe signal loss due to transverse magnetization dephasing or relaxation effects in the following sections.

3.4 Brief radio-frequency pulses are applied

Once the subject is in the magnet we have a net macroscopic magnetization that is aligned with the main field $\mathbf{B_0}$ and therefore is constant. This gives no signal, so if we want to measure the magnetization we have to do more.

3.4.1 $\mathbf{M_0}$ is excited with an RF pulse: MR signal

To measure a signal we need a transverse magnetization. Why? The equation of motion of the magnetization tells us that, if we somehow take $\mathbf{M_0}$ away from the z-axis we will create a net transverse magnetization that will precess about the direction of the main field $\mathbf{B_0}$. [To see this you can consider the rotating frame formalism, set an initial transverse magnetization M_{xy} and $B_{0(t)}$ =0, giving a constant transverse

magnetization rotating at ω_0.] A rotating magnetization generates an oscillating electromagnetic field. This field can induce a voltage across the ends of a properly designed antenna, also called a receiver coil. Such a voltage constitutes the MR signal.

So how do we rotate $\mathbf{M_0}$ off the z-axis to produce a transverse magnetization? Again, if we consider the equation of motion of $\mathbf{M}$ in the rotating frame we get the answer: we just need magnetic field $B'_{1(t)} = B'_1$ constant. If such a field is applied during a time Δt then the vector $\mathbf{M}$ will precess an angle $\Delta\theta$ about the direction of $\mathbf{B}_1$, where $\Delta\theta/\Delta t = \gamma B_1$. After the time Δt the field $\mathbf{B'}_1$ can be turned off and the magnetization $\mathbf{M}$ will remain precessing about $\mathbf{M_0}$ with a cone defined by the angle $\Delta\ \theta$ (also called the flip angle). Clearly, a flip angle of 90^o will produce the maximum transverse magnetization for any given longitudinal magnetization. Since the transverse magnetization defines the MR signal, a 90^o pulse will generate the maximum signal from a spin system at thermal equilibrium.

A constant field $\mathbf{B'}_1$ in the rotating frame means a field $\mathbf{B}_1$ oscillating at the resonance frequency in the laboratory frame. This field is called the radio-frequency (or RF) pulse and is applied by a RF coil which is designed be optimally tuned at the absorption frequency ω_0. In the MR jargon we have described a $\Delta\theta$-degrees RF pulse. In many applications the RF coil used to excite the magnetization (transmit coil) is the same coil used to detect the MR signal coming from the precessing transverse magnetization (receiver coil). Separate coils for transmitting and receiving energy are also possible, as well as arrays of receiver coils which detect signals from different volumes in the sample/subject.

The rotation angle can be adjusted by varying the pulse duration and the pulse strength. Using geometrical considerations it is not difficult to see that the most effective way (minimum pulse duration and pulse strength) of producing an arbitrary rotation is by having $\mathbf{B'}_1$ oriented perpendicular to the main field $\mathbf{B_0}$.

3.4.2 Relaxation of the MR signal

From what we have described so far it may seem that after the RF pulse is applied we will have a transverse magnetization precessesing forever about the main field direction. This is not quite the case because we have neglected relaxation effects in the equation of motion. Relaxation mechanisms make the system return to its initial equilibrium state. Thus, after an RF pulse the longitudinal component of the magnetization (M_z) will return to $\mathbf{M_0}$ and the net transverse magnetization (M_{xy}) will return to zero. The decay of the transverse magnetization is observed as signal loss (free induction decay or FID) after the RF excitation. These are two relaxation mechanisms with characteristic relaxation times T_1 and T_2, respectively.

T_1 and T_2 relaxation are complex processes that depend mostly on magnetic interactions between water molecules and other molecules. All molecules are in a constant state of motion and each of these molecules has its own magnetic field. This means that the local magnetic field experienced by a proton will be fluctuating due to the magnetic interactions with the nearby molecules. These magnetic field fluctuations, depending on both the rate and direction at which they occur can promote T_1 and T_2 relaxation.

Although the sources these relaxation mechanisms are complicated and diverse, the processes can be viewed as the response of the magnetization to randomly fluctuating local magnetic fields.

$\mathbf{T_1}$ relaxation

T_1 relaxation refers to the processes that bring the longitudinal magnetization M_z back to its equilibrium state $\mathbf{M_0}$. T_1 relaxation involves the release of excess energy, absorbed by the spins from the RF pulse, to the molecular environment, or lattice. Therefore, it is also called spin-lattice relaxation. To give away the energy there will be transitions between the populations of spins states. A change in their relative populations will affect the longitudinal magnetization and therefore T_1 relaxation is observed as the return to equilibrium of the longitudinal magnetization. T_1 relaxation is generally exponential and described by the equation:

$$\frac{dM_z}{dt} = \frac{M_0 - M_z}{T_1} \tag{11}$$

What kind of molecular mechanisms can induce T_1? We have already seen that the longitudinal magnetization can be altered by the application of a resonant B_1 field in the xy-plane. Therefore, any fluctuating magnetic field that has a component in the xy-plane that oscillates at the resonant frequency can also induce transitions between the spin states.

Typical T_1 relaxation times for water protons in biological samples are 500ms to 2000ms, with 1000ms being typical for many tissues.

$\mathbf{T_2}$ relaxation
T_2 relaxation refers to the processes that bring the transverse magnetization M_{xy} back to its value at the equilibrium state, i.e., zero. T_2 relaxation relates to the incoherent exchange of energy among neighboring spins. Because of this it is also called spin-spin relaxation. T_2 relaxation is generally exponential and described by the equation:

$$\frac{d\mathbf{M_{xy}}}{dt} = -\frac{\mathbf{M_{xy}}}{T_2} \tag{12}$$

The process of T_2 relaxation reflects the dephasing of the transverse magnetization from the different nuclei after the RF pulse. The dephasing occurs due to local magnetic field inhomogeneities in the net main field. Ideally the main field should be $\mathbf{B_0}$ for all nuclei, which will also mean that all spins will have the same precession frequency for their transverse magnetization, namely ω_o ($=\gamma B_0$). However, there will be fluctuations in the longitudinal component of the local main field, and consequently in the resonance frequencies. These fluctuations occur due to both magnetic interactions between the nuclei and imperfections in the main field homogeneity. If the spins have slightly different resonance frequencies it means that, after the RF pulse, the transverse magnetization of some spins will precess faster (where the net field $B > \mathbf{B_0}$), whereas for other spins the precession will be slower (where the net field $B < \mathbf{B_0}$). We can visualize the effect of this in the rotating coordinate system: the magnitude of the net transverse magnetization vector will get smaller with time as the individual components from different spins get out of phase from each other.

$\mathbf{T_2^*}$ relaxation
The local magnetic field inhomogeneities noted previously can be produced by two factors: 1) microscopic effects due to magnetic interactions among neighboring molecules or 2) macroscopic effects due to spatial variation of the external magnetic field. Dephasing due to molecular interactions alone is called T_2. Dephasing produced by both factors taken together is termed T_2^*. In areas where there are high magnetic field inhomogeneities T_2^* will be much shorter than T_2 resulting in a more rapid signal loss. As will be discussed further on, gradient-echo sequences, which use only one RF pulse, are sensitive to T_2^* effects. However, in spin-echo sequences, the application of an additional 180-degree RF pulse after excitation results in the formation of a signal (the so-called spin-echo) that is primarily sensitive to T_2 rather than T_2^* effects.

3.4.3 Bloch equations

The differential equation that describes the dynamics of the magnetization in the presence of an external magnetic field (8) can be combined with the relaxation terms (11 and 12) into one vector equation:

$$\frac{d\mathbf{M}_{(t)}}{dt} = \mathbf{M}_{(t)} \times \gamma \mathbf{B}_{ext(t)} + \frac{1}{T_1}(M_0 - M_z)\hat{z} - \frac{1}{T_2}\mathbf{M_{xy}} \tag{13}$$

This empirical vector equation is referred to as the Bloch equation. The relaxation terms describe the return to equilibrium, but only for a field pointing along the z-axis. These equations can be solved under a variety of boundary conditions to describe the time domain of the NMR signal following a stimulus.

3.5 Summary

The following is a list of concepts that should be clear by the end of this lecture.

- Spinning protons, frequently referred to as just spins, can be represented by a small magnetization vector.
- A slight excess number of protons align with the main external magnetic field $\mathbf{B_0}$. The number in excess is proportional to the magnitude of $\mathbf{B_0}$. The total magnetic field of the excess protons is the equilibrium magnetization $\mathbf{M_0}$.

- Spinning protons wobble or precess about the external field $\mathbf{B_0}$. The resonance frequency is called the Larmor frequency ω_0 and is proportional to $\mathbf{B_0}$.
- Radio frequency pulse: a magnetic field with a transverse component oscillating at the Larmor frequency can be used to rotate the magnetization vector. If a RF pulse with flip angle α is applied to the equilibrium magnetization $\mathbf{M_0}$, then after the RF pulse the new longitudinal and transverse components will be $M_z = M_0 \cos\alpha$ and $M_{xy} = M_0 \sin\alpha$, respectively.
- NMR signal: a transverse component M_{xy} will precess about $\mathbf{B_0}$ with the Larmor frequency, thus inducing a measurable signal in a properly tuned coil.
- Laboratory Frame: The viewpoint of an observer in the laboratory. The laboratory is stationary, the protons are spinning.
- Rotating Frame: The viewpoint of an observer riding along on the protons. The protons appear stationary, the laboratory is rotating. The rotating frame is a convenient reference frame to visualize manipulations of the magnetization by externally applied RF pulses.
- Spin-Lattice Relaxation: The process whereby energy absorbed by the excited protons or spins is released back into the surrounding lattice, reestablishing thermal equilibrium.
- T_1 Relaxation: Spin-Lattice relaxation. The exponential recovery of longitudinal (aligned with $\mathbf{B_0}$) magnetization. M_z returns to $\mathbf{M_0}$.
- Spin-Spin Relaxation: The temporary and random interaction between two excited spins that causes a cumulative loss in phase resulting in an overall loss of signal. Also known as transverse or T_2 relaxation.
- T_2 Decay: The exponential loss of signal resulting from purely random spin-spin interactions in the transverse or XY plane.
- Bloch Equations: these are equations that describe the dynamics of the magnetization vector under the influence of externally applied magnetic fields (e.g., the static field $\mathbf{B_0}$ and the fields from RF pulses) in addition to the relaxation effects.

4 Bibliography

- Haacke, Brown, Thompson and Venkatesan (1999) *Magnetic Resonance Imaging: Physical Principles and Sequence Design* (Chapt. 1-4), Wiley-Liss.